J. Dr[illegible] E[illegible] A[illegible] [illegible]

Geometric Transient Solutions of Autonomous Scalar Maps

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[a] [illegible]
[b] [illegible]

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1. THE PROBLEM

[illegible]

$$x(t+\square) = f(x(t)) \qquad (\square)$$

[illegible]

$$x(t) = \sum_{n=\square}^{\infty} c_n r^{nt}, \quad c_\square = x_e$$

* [illegible]
† [illegible]

[illegible]

[illegible] < | | < [illegible]? [illegible]

[illegible]?

[illegible] + [illegible]

[illegible] = [illegible]

[illegible]

$$x(t+[illegible]) = f(a, x(t)) = ax(t) + h(a, x(t)) \qquad ([illegible])$$

[illegible]

$$h(a,x) = \sum_{i=[illegible]}^{\infty} \frac{[illegible]}{i!}\frac{\partial^i f}{\partial x^i}(a,[illegible])x^i.$$

[illegible] $\mathbf{x} \in {}_{+} \times H$ [illegible] $\mathbf{x} = \{x(t)\}_{t=[illegible]}^{\infty} \in H_r$ [illegible] $\mathbf{x}_e = \{x_e\}_{t=[illegible]}^{\infty}$ [illegible]

[illegible] $\mathbf{0}$ [illegible] $\mathbf{0} = \{[illegible]\}_{t=[illegible]}^{\infty}$ [illegible] $\in {}_{+}$ [illegible] ${}_{+} \times H$ [illegible] $<$ [illegible] $>$ [illegible] $\mathbf{x}$ [illegible] $\mathbf{0}$ [illegible] $>$ [illegible]

[illegible] $\lim_{a \to [illegible]^{+}} x_e(a) = [illegible]$

[illegible]

2. LINEAR THEORY

[illegible]

$$x(t + [illegible]) = ax(t) \qquad ([illegible])$$

$$x(t + [illegible]) = ax(t) + b(t) \qquad ([illegible])$$

[illegible]

$$b(t) = \sum_{n=[illegible]}^{\infty} d_n r^{nt}$$

[illegible] H [illegible] E [illegible] $\neq$ [illegible] $\in$ [illegible] H [illegible] H [illegible] H [illegible]

$$x([illegible]) = \sum_{n=[illegible]}^{\infty} \frac{d_n}{r^n - a};$$

[redacted] $\in$ [illegible] H [illegible]

[illegible] $\neq$ [illegible] $\in$ [illegible] H [illegible]

$$
\begin{aligned}
x(t) &= a^t x(\square) + a^{t-\square} \sum_{n=\square}^{\infty} d_n \sum_{i=\square}^{t-\square} \left(\frac{r^n}{a} \right)^i \\
&= a^t x(\square) + a^{t-\square} \sum_{n=\square}^{\infty} d_n \frac{\square - (r^n/a)^t}{\square - (r^n/a)} \\
&= a^t \left(x(\square) - \sum_{n=\square}^{\infty} \frac{d_n}{r^n - a} \right) + \sum_{n=\square}^{\infty} \frac{d_n}{r^n - a} r^{nt}
\end{aligned}
$$

[illegible] H

$\mathbf{v} = \{r^{mt}\}_{t=}^{\infty} \in H_r$

$$P\mathbf{b} = \mathbf{b} - \langle \mathbf{b}, \mathbf{v} \rangle \mathbf{v}$$

$\mathbf{b} \in H$ … $\mathbf{b} + (I - P)\mathbf{b}$ …

3. NONLINEAR THEORY

$$x(t+1) = f(a, x(t)) = ax(t) + h(a, x(t))$$

3.1. Bifurcations from Zero

$$x(t+1) - r^m x(t) = (a - r^m)x(t) + h(a, x(t)).$$

[illegible] $H \longrightarrow H$ [illegible]

$$L\{x(t)\}_{t=\square}^{\infty} = \{x(t+\square) - r^m x(t)\}_{t=\square}^{\infty}$$

[illegible] B [illegible] $H \longrightarrow H$ [illegible]

$$B(a, \{y(t)\}_{t=\square}^{\infty}) = \{(a - r^m)y(t) + h(a, y(t))\}_{t=\square}^{\infty}.$$

[illegible] $\mathbf{v} = \{r^{mt}\}_{t=\square}^{\infty}$ [illegible]

[illegible]

$$L\mathbf{x} = B(a, \mathbf{x})$$

[illegible] $\mathbf{x} \in H$ [illegible] $=$ [illegible]

[illegible]

$$\mathbf{x} = \varepsilon\mathbf{v} + \varepsilon\mathbf{w}($$

[illegible] $(\varepsilon, \lambda, \mathbf{w}) \in$ [illegible]

$$\mathbf{0} = \mathbf{w}(\varepsilon) - L^{-1}PT(\varepsilon, \lambda, \mathbf{w})$$

$$\mathbf{0} = (I - P)T(\varepsilon, \lambda, \mathbf{w}).$$

[illegible] $H = \ldots \oplus \ldots$ [illegible] $\lambda(\varepsilon)$ [illegible] $\mathbf{w}(\varepsilon)$ [illegible] ε.

[illegible] $\Gamma : \ldots \times \ldots \times H \longrightarrow H$ [illegible]

$$\Gamma(\varepsilon, \lambda, \mathbf{w}) = (I - P)T(\varepsilon, \lambda, \mathbf{w}).$$

[illegible] Γ [illegible] $\mathbf{0}) = \mathbf{0}$ [illegible] (ε) [illegible] Γ [illegible] λ [illegible] $\mathbf{0}$ [illegible]

$$\Delta\lambda \longmapsto \{r^{mt}\Delta\lambda\}_{t=0}^{\infty}$$

[illegible] $\lambda = \lambda(\varepsilon, \mathbf{w})$ [illegible] $\lambda(\mathbf{0}) =$ [illegible] $\lambda(\varepsilon, \mathbf{w})$ [illegible]

$$\mathbf{0} = \mathbf{w} - L^{-1}T(\varepsilon, \lambda(\varepsilon, \mathbf{w}), \mathbf{w}).$$

[illegible] $\mathbf{w} = \mathbf{w}(\varepsilon)$ [illegible] ε [illegible] $\lim_{\varepsilon \to 0} \mathbf{w}(\varepsilon) = \mathbf{0}$.

[illegible]

[illegible] A [illegible] (A) [illegible] F [illegible] $\in$ [illegible] $\in$ [illegible] ε [illegible] (ε) [illegible] $(\varepsilon)) \in \ldots_+ \times H$ [illegible] $= B$ [illegible] $\mathbf{0})$ [illegible] $\varepsilon \to 0$ [illegible] (ε) [illegible] $(\varepsilon)) =$ [illegible] $\mathbf{0})$.

[illegible] ε [illegible] $(a(\varepsilon), \mathbf{x}(\varepsilon)) = (a(\varepsilon), \{x(t, \varepsilon)\}_{t=0}^{\infty})$ [illegible] $\{x(t, \varepsilon)\}_{t=0}^{\infty} \in H_r$ [illegible] $=$ [illegible] (ε).

3.1.1. D[illegible] B[illegible]

[illegible] $=$ [illegible] $\mathbf{0})$ [illegible] ε [illegible] (ε) [illegible] $(\varepsilon) <$ [illegible]

[illegible]

$$w(t + [illegible], \varepsilon) - r w(t, \varepsilon) = \lambda(\varepsilon)[r^t + w(t, \varepsilon)] + \frac{[illegible]}{\varepsilon} h(r + \lambda(\varepsilon), \varepsilon r^t + \varepsilon w(t, \varepsilon))$$

[illegible]

$$[illegible] = \lambda(\varepsilon)$$

[illegible]

[illegible] $R_+ \times H_{r^M}$ [illegible] $\mathbf{0}$

[illegible] $(a, \mathbf{x}(\varepsilon)) = (a, \{x(t,\varepsilon)\}_{t=[illegible]}^{\infty})$ [illegible]

[illegible]

3.2. Bifurcations from the Positive Equilibrium Branch

[illegible] $\mathbf{x} = B$[illegible] [illegible]

$$y(t + [illegible]) = \alpha y(t) + \eta(y(t)) \qquad ([illegible])$$

[illegible] $\alpha =$ [illegible]

[illegible] α [illegible] $\alpha \in$ [illegible] $\times H_\alpha$ [illegible] $\mathbf{0}$ [illegible] $\times H_\alpha$ [illegible] $\times H_\alpha$ [illegible] $\mathbf{x} = B$ [illegible] $\mathbf{x}(\varepsilon) = \mathbf{y}(\varepsilon) + \mathbf{x}$ [illegible]

[illegible] A [illegible] F [illegible] $\times H$ [illegible] $= B$ [illegible] F [illegible] ε [illegible] $\to \infty$ [illegible]

[illegible] $(a, \mathbf{x}(\varepsilon)) = (a, \{x(t,\varepsilon)\}_{t=[illegible]}^{\infty})$ [illegible] $\to \infty$ [illegible]

[illegible] H [illegible]

3.3. H-stability

[illegible]

4. EXAMPLE: CALCULATING SOLUTIONS OF THE LOGISTIC MAP

[illegible]