

Explaining and predicting patterns in stochastic population systems

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For the experimental parameters and initial condition reported in Dennis et al. (2001)— $b = 10.67$, $m_i = 0.1955$, $m_a = 0.9600$, $c_{ei} = 0.01647$, $c_{ea} = 0.01313$, $c_{pa} = 0.3500$, $V = 1$ and $[L_0, P_0, A_0] = [250, 5, 100]$ —the LPA model predicts chaos with a recurring 11-cycle pattern (Cushing et al. 2001; King et al. 2003), whereas the mode map predicts a six-cycle. The realizations of the stochastic process (2.7) contain episodes of the dynamics of the mean map as well as episodes of the dynamics of the mode map (figure 2).

3. DISCUSSION

We have illustrated how recurrent patterns in stochastic processes may be predicted by various deterministic models derived from the parent stochastic process. The mean and mode maps are examples of two such deterministic models.

The patterns predicted by the mean and mode maps are related. One can the

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